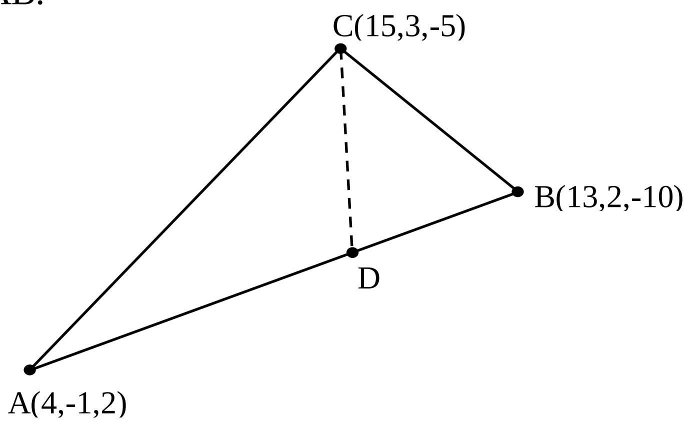
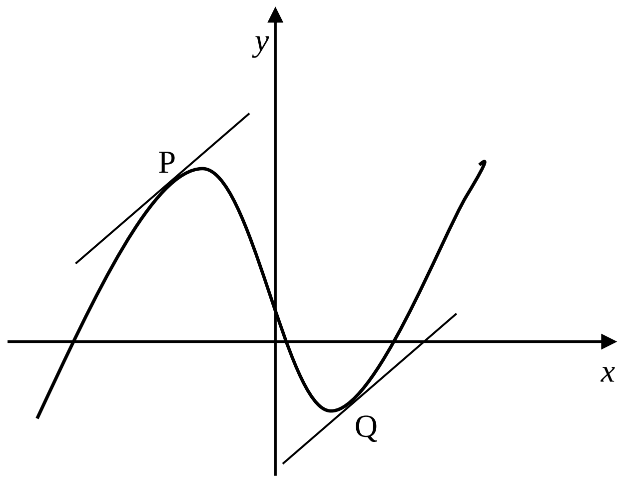
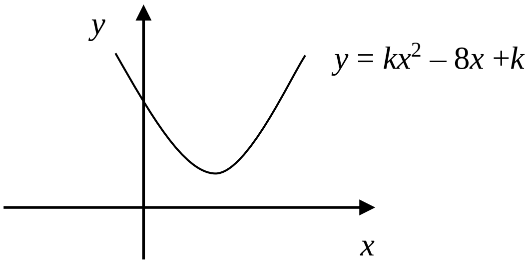
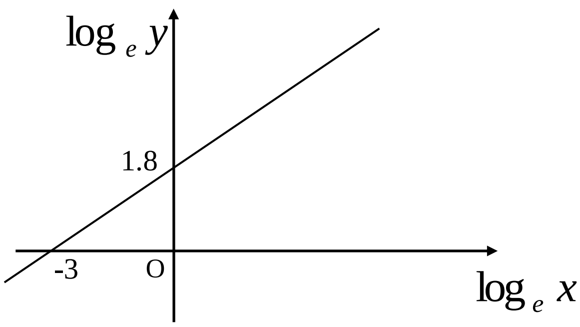
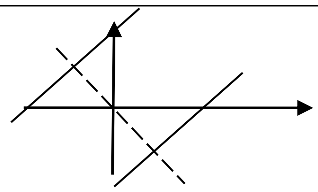
	Higher Prelim Revision 4 – Paper 1 Non-Calculator	**30**
1.	A line segment joins points A(2,2) and B (8,6). Find the equation of the perpendicular bisector of line AB	**3**
2.	A and B are acute angles such that $\tan A = \frac{\sqrt{7}}{2}$ Find the exact value of (i) Sin 2A (ii) Cos 2A	**3**
3.	(a) Express $x^4 - x$ in its fully factorised form (b) Hence or otherwise state the coordinates of the x and y-intercepts for the curve $y = x^4 - x$	**5** **1**
4.	Solve $2\sin 3x - 1 = 0$ in the interval $0 \leq x \leq 180°$	**3**
5.	Differentiate $\sin(6x^2)$ with respect to x	**2**
6.	Solve $\log_3(x+1) + \log_3(x-1) = 1, \quad x > 1$	**4**
7.	For all points on the curve $y = f(x)$, $f'(x) = x^2 - 6$ If the curve passes through the point (3,-5), find the equation of the curve	**4**
8.	(a) Show that $(\cos x + \sin x)^2 = 1 + \sin 2x$ (b) Hence find $\int (\cos x + \sin x)^2 \, dx$	**2** **3**

	answers
1.	Midpoint of AB is (5,4), $m_{AB} = 2/3$, $m_{perp} = -3/2$, equation of perpendicular bisector is $y - 4 = -3/2(x - 5)$ or **$2y + 3x = 23$**
2.	sinA = $\frac{\sqrt{7}}{\sqrt{11}}$, cosA = $\frac{2}{\sqrt{11}}$ sin2A = 2sinAcosA = $2\times\frac{\sqrt{7}}{\sqrt{11}}\times\frac{2}{\sqrt{11}}=\frac{4\sqrt{7}}{11}$ cos2A = $2\cos^2$A - 1 = $2\times\left(\frac{2}{\sqrt{11}}\right)^2-1=-\frac{3}{11}$
3.	$x^4 - x = x(x^3 - 1)$ using synthetic division for $x^3 - 1$ $\begin{array}{r\|rrrr} 1 & 1 & 0 & 0 & -1 \\ & 0 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$ No remainder, $(x - 1)$ is a factor $x(x^3 - 1)$ = **$x(x - 1)(x^2 + x + 1)$** fully factorised intercepts are **(0,0) and (1,0)**
4.	$\sin 3x = ½$ → $3x$ = 30°, 150°, 390°, 510° → **x = 10°, 50°, 130°, 170°**
5.	$\frac{dy}{dx}=\cos(6x^2)\times 12x \;=12x\cos(6x^2)$
6.	$\log_3(x+1)(x-1)=1 \rightarrow (x+1)(x-1)=3^1 \rightarrow x^2-1=3 \rightarrow x^2=4 \rightarrow x=\pm 2$ **$x > 1$, so $x = 2$**
7.	$y=\int x^2-6\,dx \rightarrow y=\frac{1}{3}x^3-6x+c \rightarrow -5=\frac{1}{3}(3)^3-6(3)+c \rightarrow c=4$ $y=\frac{1}{3}x^3-6x+4$
8a	$(\cos x+\sin x)^2=(\cos x+\sin x)(\cos x+\sin x)$ $=\cos^2 x+2\sin x\cos x+\sin^2 x$ $=\cos^2 x+\sin^2 x+2\sin x\cos x$ $=1+2\sin x\cos x$ $=1+\sin 2x$ as required
8b	$\int(\cos x+\sin x)^2\,dx = \int 1+\sin 2x\,dx$ $= x-\frac{1}{2}\cos 2x+c$

	Higher Prelim Revision 4 – Paper 2 Calculator	**40**
1.	Triangle ABC has vertices A(4,-1,2), B(13,2,-10) and C(15,3,-5) as shown. Point D lies on side AB. 	
	(a) Given that D divides the line AB in the ratio 2:1, show that D has coordinates (10, 1, –6).	**3**
	(b) Hence calculate the size of angle CDB.	**5**
2.	The diagram shows a sketch of the graph $y = x^3 - 9x + 4$, with two parallel tangents drawn at P and Q 	
	(a) Find the equations of the tangents to the curve $y = x^3 - 9x + 4$ which have a gradient of 3	**6**
	(b) Show that the shortest distance between the tangents is $\frac{16\sqrt{10}}{5}$	**5**

3.	Calculate the value of k so that the graph of $y = kx^2 - 8x + k$ does not cut or touch the x-axis  	**4**
4.	Given that two functions are defined by $f(x) = 2x - 1$ and $g(x) = x^3$, evaluate $$\int_1^2 \left[g(f(x)) \right] \, dx$$	**4**
5.	A curve has equation y = 7sin$x$ – 24cos$x$	
	(a) Express in the form ksin(x - a), $0 \le x \le \pi/2$	**4**
	(b) Hence find, in the interval $0 \le x \le \pi$, the x-coordinate of the point on the curve where the gradient is 1	**3**
6.	The results of an experiment give the graph shown below  	
	(a) The graph passes through the points (-3,0) and (0, 1.8) Write down the equation of this line in terms of $\log_e x$ and $\log_e y$	**2**
	(b) Show that x and y satisfy a relationship in the form $y = ax^b$, stating the values of a and b	**4**

Answers	
1a	$\underline{\mathbf{d}} = \underline{\mathbf{a}} + \tfrac{2}{3}\overrightarrow{AB} \longrightarrow \underline{\mathbf{d}} = \begin{pmatrix}4\\-1\\2\end{pmatrix},+\frac{2}{3}\begin{pmatrix}9\\3\\-12\end{pmatrix}=\begin{pmatrix}10\\1\\-6\end{pmatrix} \longrightarrow$ D = (10, 1, -6) as required
1b	$\cos CDB = \dfrac{\overrightarrow{DC}\bullet\overrightarrow{DB}}{\lvert\overrightarrow{DC}\rvert\lvert\overrightarrow{DB}\rvert}$, $\overrightarrow{DC}=\begin{pmatrix}5\\2\\1\end{pmatrix}$, $\overrightarrow{DA}=\begin{pmatrix}3\\1\\-4\end{pmatrix}$ $\overrightarrow{DC}\bullet\overrightarrow{DB}=13$ $\lvert\overrightarrow{DB}\rvert=\sqrt{30}$ $\lvert\overrightarrow{DC}\rvert=\sqrt{26}$ $\cos CDB=\dfrac{13}{\sqrt{26}\sqrt{30}}$, **angle CDB is 62°**
2a	$\dfrac{dy}{dx}=3x^2-9= \longrightarrow 3x^2-9=3 \longrightarrow 3x^2-12=0 \longrightarrow 3(x-2)(x+2)=0$ P(-2, 14), Q(2, -6) **Tangent at P is $y = 3x + 20$ Tangent at Q is $y = 3x - 12$**
2b	Shortest distance is along a line perpendicular to both tangents with a gradient of $-\tfrac{1}{3}$ Equation of perpendicular line is  $y = -\tfrac{1}{3}x$ Point of intersection with tangent through P is $-\tfrac{1}{3}x = 3x + 20$, $10x = -60$ (-6, 2) Point of intersection with tangent through Q is $-\tfrac{1}{3}x = 3x - 12$, $10x = 36$ (18/5, -6/5) Distance between these points is $\sqrt{\left(-6-\frac{18}{5}\right)^2+\left(2+\frac{6}{5}\right)^2} = \dfrac{16\sqrt{10}}{5}$ as required
3.	For the quadratic not to touch the x-axis, $b^2 - 4ac < 0$ $64 - 4k^2 < 0 \longrightarrow 4(4-k)(4+k) < 0 \longrightarrow$ **$k < -4$, $k > 4$**
4.	$g(f(x))=(2x-1)^3$, so $\int_1^2 (2x-1)^3\,dx = \left[\dfrac{(2x-1)^4}{8}\right]_1^2 = \left(\dfrac{(2(2)-1)^4}{8}\right)-\left(\dfrac{(2(1)-1)^4}{8}\right)=10$
5a	$k\sin x\cos a - k\cos x\sin a$, $k\cos a = 7$, $k\sin a = 24$, $k = 25$, $a = 1.29$ **25sin(x - 1.29)**
5b	$\dfrac{dy}{dx}=1= \longrightarrow 25\cos(x-1.29)=1 \longrightarrow x-1.29 = 1.53,\ 4.75$ **x = 2.82** , $\cancel{6.04}$
6a	$\log_e y=\dfrac{3}{5}\log_e x+1.8$
6b	$y=ax^b \longrightarrow \log_e y=\log_e ax^b \longrightarrow \log_e y=\log_e a+\log_e ax^b \longrightarrow \log_e y=\log_e a+b\log_e x$ From part (a) $b=\dfrac{3}{5}$ and $\log_e a = 1.8 \rightarrow a=e^{1.8}$ $y=e^{1.8}x^{3/5}$